

Teaching Mathematics for Robust Understanding

What makes a mathematically powerful classroom?

Leader's Guide

GOAL

This tool is intended to help develop an understanding of the Teaching for Robust Understanding (TRU) framework for Mathematics as a foundation for thinking about, planning, observing, and reflecting on classroom teaching.

USERS

This introduction can be used with a wide variety of groups, including: district personnel who are in a position to coach, observe, or otherwise support teacher professional development; principals who need to provide a supportive environment for staff learning; in-service teachers at the beginning of professional development; tutors and student teachers in the early stages of a teacher preparation course. It provides a compact introduction to the suite of materials on <http://map.mathshell.org/trumath.php>

INTRODUCTION

Reflecting the standards in high-performing countries, the Common Core and related state standards embody a broader and deeper view of mathematics than has been traditional in US schools. As well as emphasizing robust understanding of mathematical *content*, it includes the *practices* of doing mathematics. This requires a deeper way of thinking about what matters in classrooms.

Teaching for Robust Understanding in Mathematics (TRU) is a research-based framework for doing this. The five dimensions of TRU are: (i) the mathematics, (ii) cognitive demand, (iii) access to mathematical content, (iv) agency, ownership and identity and (v) formative assessment. If things go well in classrooms along these five dimensions, the students who emerge will be powerful thinkers and problem solvers.

The purpose of this workshop session is to introduce participants to the framework and to some of the tools that can be used with it. By the end of the workshop we hope they will see that:

- TRU is a way of thinking, not just a set of tools. It can be used by individuals and communities in an ongoing way for planning, observing, and reflection.
- Getting better across the dimensions of TRU leads to improved instruction.
- There are no “magic bullets.” Change takes time, and TRU can help to support ongoing improvement.

The approach taken in this workshop is aimed at having participants see TRU emerge from their own thoughts about and discussions of specific examples of teaching. It gives participants a chance to react to the three stimulus videos, so that the analysis that follows is seen as pulling together their own thoughts, not imposing something from “the outside.”

SESSION OUTLINE

- | | |
|--|------------------------------|
| • What matters in classrooms? | 5 minutes |
| • Part 1: An introduction to TRU – 3 videos analyzed | 15+15 + 10 + 10 = 50 minutes |
| • Part 2: Introducing TRU Tools | 20 minutes |
| • Q & A | 15 minutes |

MATERIALS REQUIRED

- PowerPoint: TRU Slides.pptx
- Either a whiteboard large enough to record 5 columns' worth of participants' comments – or five flip charts, with markers. (See Slide 7 below)
- Session Handouts: One copy per participant.

TIME NEEDED

90 minutes, with opportunities for expansion.

PREPARATION

Please work through this guide carefully, referring to the Handouts and Powerpoint. The core Activity Sequence (below) covers the same material as on the PowerPoint slides, including the notes with each slide, distinguishing suggested comments and *instructions*. Fill in your local information on the first and last slides.

Part 1 is intended to go slowly. Participants will generate the ideas that you, the leader, synthesize. This part is interactive. Be sure that you get comfortable with TRU yourself so you can recognize which of its dimensions each comment from the participants in Part 1 fits best.

Part 2 is more of a presentation, where three tools are presented. The idea is to zip through things, taking time to linger on just one page as an example.

Try to anticipate the common issues that participants will have and note down your responses to them, in the table below. The examples shown were taken from trials of this session.

Common concern

Suggested responses

<ul style="list-style-type: none">• It seems awfully complicated, with its five dimensions. I thought teaching was about explaining the math, showing how each procedure works, and having the students practice until they know it.	<ul style="list-style-type: none">• That's a traditional approach and it works with some students - but you know that a lot of students, even those who 'get it' at the time, can't do it six months later. That's why TRU is about "<i>robust understanding</i>" - which depends on students getting to grips with the math more deeply (cognitive demand) and making it their own not just the teacher's (agency, ownership, identity)
<ul style="list-style-type: none">• Doesn't teaching like this take a lot more time?	<ul style="list-style-type: none">• It does, but it saves a lot of re-teaching topics - and re-teaching, usually faster, is not an effective approach to remediation.
<ul style="list-style-type: none">•	<ul style="list-style-type: none">•
<ul style="list-style-type: none">•	<ul style="list-style-type: none">•

ACTIVITY SEQUENCE

Title Slide

You may like to customize this slide and/or the last one with your own institutional and contact details. Please leave the copyright attribution, however.

Possible comments below are in plain text. Suggestions are in italics.

Users will, of course, adapt as necessary – though we recommend sticking with this activity sequence the first time or two.

Introduce the session by saying (in your own words) that the purpose of the session is to provide participants with a deep sense of the Teaching for Robust Understanding (TRU) framework.

Slide 1

Mathematics Assessment Project



Teaching for Robust Understanding
What makes a mathematically powerful classroom?

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Introduction (5 minutes)

In Part 1 we develop a framework that seeks to answer the question: What really matters in classrooms?

We will do this by watching three videos and discussing what the lesson is trying to achieve.

The framework will emerge from your conversations.

In part 2 I'll introduce some tools that help to support powerful classroom instruction.

Then we'll close with questions and answers.

Slide 2

Today's Agenda

1. What really matters in classrooms?
2. Some tools for supporting powerful classroom instruction:
 - Formative Assessment lessons
 - Planning and Reflection
 - Classroom Observation Rubric
3. Q&A, on anything you want to talk about

1. What matters in classrooms?

Show this slide quickly, then move on.

Slide 3



1. What really matters in classrooms?

Give people a bit of time to discuss and note down answers to these two questions.

Note your ideas on **Handout 1**.

You may well ask: why just five?

- It's as many as most folks can keep in mind. (In fact, it may be too many to work on at one time.)
- If you have 20, you might as well have none. People can't keep that many things in their heads, and long checklists don't help. What matters is what people can act on, in teaching and coaching.

Slide 4

Establishing a focus

- If you had 5 things to focus on in order to improve mathematics teaching, what would they be?
- How would you know they're the right things?



1. The angles lesson

(~15 minutes after 5 minutes)

We'll now talk about what you see in each of three videos. The TRU framework will emerge from your observations.

Feel free to compare and contrast – but make sure to focus on what the experience looks like from the point of view of the students.

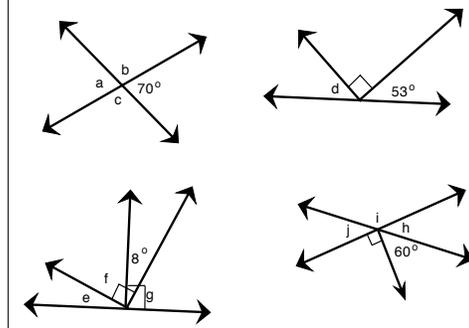
The first lesson we'll look at is the TIMSS Geometry Video on Angles.

This is the task that students are looking at. Think about what it involves.

Slide 5

1. A lesson on finding angles

Find the measure of all angles



On your own, note down what you notice about the lesson on **Handout 2**

What is it like to be a student in this classroom?

Play the video. (4 minutes).

This video is taken from:

<http://www.timssvideo.com/videos/mathematics/United%20States>

Slide 6

1. A lesson on finding angles



You need to mark out 5 columns on the whiteboard, (or have 5 flip charts set up).

Complete your notes on **Handout 2**

Then share your observations with a partner.

Give the group a minute to think then 1-2 minutes to talk in 2s or 3s.

Now collect a few comments on the whiteboard or the flipcharts.

Have the 5 TRU dimensions in your head, but do not write them on the whiteboard yet. Leave room at the top for putting the headings after you've looked at all three videos.

When a comment about **mathematics** is made, write it in the first column. Comments about **cognitive demand** go in column 2, **equity** column 3, **agency and identity** column 4, and **formative assessment** column 5. **Don't worry about being too accurate – it all works out in the end.**

Don't say anything about why; just move to the second video after a few minutes.

Slide 7

Think, Pair, Share



- What did you notice?
- What was the experience like for the students?

Vocabulary and small calculations of angles	Demand is low. Math is in bite-sized pieces. No thinking time.	Participation is limited – not much to think about.	No discussion, students sit apart.	Understanding is not revealed. Questioning is closed, and doesn't require explanations.
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2. The border tiles lesson

(15 minutes after 20 minutes)

The set-up is important here.

Explain the problem and remind the participants to think about what it looks like from the student's perspective.

Toward the end of the tape tell them that next steps include thinking about the same problem with a 6 x 6 square, and then an $n \times n$ square.

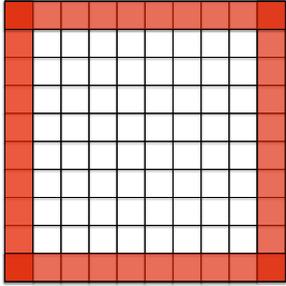
Slide 8

2. The Border Tiles Lesson

Here's a 10 x 10 grid.

Without counting them all one by one, how can you figure out the number of red border tiles?

What about other sizes?



Again, make notes on what you see on Handout 2

Play the video (6 minutes)

From Boaler and Humphreys (2005)
*Connecting Mathematical Ideas: Middle School
Video Cases to Support Teaching and Learning*

Slide 9



Now we'll reflect on this lesson as we did before.
Write additional comments on the whiteboard using
a different color.

Work through enough comments to get the sense of
how different this tape is from the first.
Comparative comments are fine.

(Key points to keep in mind when facilitating the
comments: the math starts out simple but gets
interesting – will all the formulas give the same
answer? Student thinking is the focus, and it's not
trivial; there is some challenge. Lots of kids
participate, and they're invited in; they get to talk
math, and their thinking is fully heard.)

There is no need to say this yourself, but when you
paraphrase as you write things down, these brief
descriptions might be useful.

Slide 10

Think, Pair, Share



- What did you notice?
- What was the experience like for the students?

3. The fractions, decimals, percents lesson

(10 minutes after 35 minutes)

This lesson was with a 6th grade classroom in an
inner-city, low income school in Chicago. The
lesson uses materials from the project website:

<http://map.mathshell.org/>

It starts out with their cutting out the cards and
filling in the blanks – they have to write in the % if
the decimal is given, and vice-versa.

Then they have to order them.

It's essential that they collaborate.

Notice that one card has no numbers on it.

Slide 11

3. Fractions, decimals percents

Take turns to:

1. Fill in the missing decimals and percents.
2. Place the cards in order of size.
3. Check that you agree.

0.2 _____ ____%	0.05 _____ ____%	_____ _____ 80%
0.375 _____ ____%	_____ _____ 12.5%	0.75 _____ ____%
1.25 _____ ____%	_____ _____ 50%	_____ _____ ____%

Move quickly through to the video – this is just to give a picture of the full lesson

In the full lesson, after they do decimals and percents, the students are given other representations and have to match the cards.

First they are given area diagrams.

Then they are given fraction representations and scales.

As cards are placed, they are again ordered.

Students are encouraged to collaborate on this, detecting and correcting errors, and filling in the gaps.

The slide shows the answer set. THE gray squares indicate blank cards filled in by the students.

Slide 12

3. Fractions, decimals percents

0.05 5%	0.125 12.5%	0.2 20%	0.375 37.5%	0.5 50%	0.6 60%	0.75 75%	0.8 80%	1.25 125%
$\frac{1}{20}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{6}{10}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{5}{4}$

The gray cards are the ones that students had to create for themselves.

Now for a video episode in a class working on the first set of cards.

Again note what you observe on Handout 2

Play the video. (3 minutes)

Slide 13

3. Fractions, decimals percents

Again give time for people to complete their notes.

Now that this is the last video, give a little more time in the “share” portion for people to say what they want, including comparative comments.

You’re still organizing them in 5 columns, without headings.

Slide 14

Think, Pair, Share

- What did you notice?
- What was the experience like for the students?

Recognizing TRU

(10 minutes after 45 minutes)

Now let me show you how I organized your comments.

At this point write the headings on top of each of the columns (or flip charts). As you do, read through the questions on the slide.

The slide shows a brief summary of the TRU framework. It's on **Handout 3**.

So what's new or different? In a sense, nothing. You should recognize and resonate to everything in TRU. It captures what we know is important. It doesn't offer any "magic bullets" or surprises.

Alan Schoenfeld has led the team, who have been working for many years on TRU. As I play the audio file, Alan will introduce us to the framework.

As Alan speaks, try to relate the foci of your classroom observations to the various categories in his framework.

Click on the picture of Alan to hear the audio file. (4 minutes)

TRU Math was developed to highlight critical components of teaching and learning. It provides a means of structuring our observations and discussions with teachers.

There's one problem with what you've seen thus far. Text is linear, but the ideas aren't. So here's a re-framing.

The math is at the heart of everything; and everything is connected, either by adjacency or across. For example, access to the math is a key ingredient on the way to developing agency, but you get it by working at the right level of cognitive demand ("productive struggle"), which is best set if you have a chance to hear what students think and adjust instruction accordingly (formative assessment). So, it's all connected, BUT, you can take it apart to work on any dimension you want to focus on.

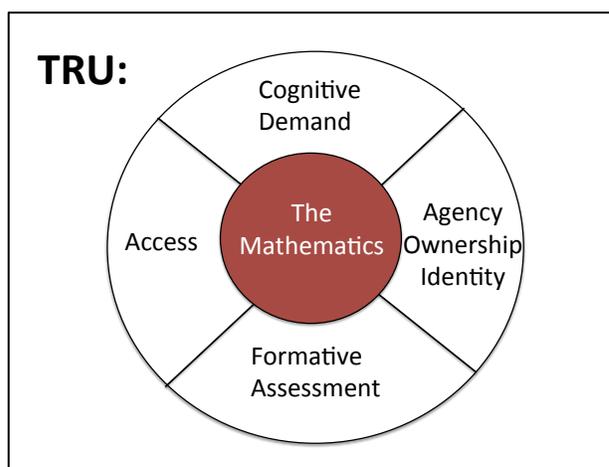
Slide 15

These were the headings	
The Mathematics	<ul style="list-style-type: none">Is it important, coherent, connected?Opportunities for thinking and problem solving?
Cognitive Demand	<ul style="list-style-type: none">Do students have opportunities for sense making?Do they engage in productive struggle?
Access and Equity	<ul style="list-style-type: none">Who participates in what ways?Do <i>all</i> students engage in sense-making?
Agency, Ownership	<ul style="list-style-type: none">Do students have the opportunity to do and talk math?Do they come to see themselves as math people?
Formative Assessment	<ul style="list-style-type: none">Does classroom discussion reveal what students understand, so that instruction may be adapted to help students learn?

Slide 16

5 Dimensions of Mathematically Powerful Classrooms	
The Mathematics	
Cognitive Demand	
Access to Mathematical Content	
Agency, Ownership, and Identity	
Formative Assessment	

Slide 17



Alan Schoenfeld and colleagues claim that
Read the slide

But before proceeding, it's ESSENTIAL to understand:

- TRU is NOT a tool or set of tools.
- TRU is a perspective regarding what counts in instruction, and
- TRU provides a language for talking about instruction in powerful ways.

With this understanding, you can make use of any productive tools more wisely.

Evidence for the claims may be found at <http://map.mathshell.org> and <http://ats.berkeley.edu>

Part 2. Tools for supporting instruction

(20 minutes after 55 minutes)

This part – all of Part 2 – is in “show and tell” mode - in contrast to the workshop mode for Part 1.

From here on we'll briefly look at three types of tools.

Read the slide

More tools, and more information, are available at the websites (which will be shown again at the end of the workshop).

Tool 1: Formative Assessment Lessons

Classroom Challenges are lessons that support teachers in formative assessment. There are 100 lessons in total, 20 at each grade from 6 to 8 and 40 for 'Career and College Readiness' at High School Grades 9 and above. Some lessons are focused on developing math concepts, others on solving non-routine problems. They each take about 2-3 class periods.

The goal here is to take a high speed tour through one FAL and show how well the FALs support teaching that's in line with TRU.

Slide 18

Our claim

Research suggests that:

- Classrooms at all grade levels that do well along these five dimensions will produce students who are powerful mathematical thinkers.
- Instructional materials, professional development, and classroom observations will be most powerful if they are aligned with these five dimensions.

Slide 19



2. Tools for supporting powerful classroom instruction

- a. Formative Assessment lessons
- b. Planning and Reflection
- c. Ways to observe classrooms

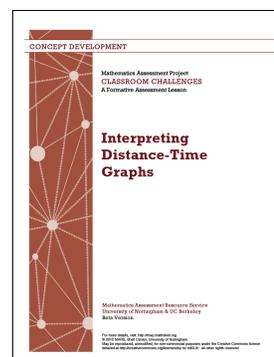
Slide 20

Tool (a) Formative Assessment Lessons

<http://map.mathshell.org>

- 100 Classroom Challenges:
- 20 at each grade 6-8
 - 40 for Career and College readiness at High School.
-
- 70% concept focused
 - 30% problem solving

Each lesson attempts to integrate formative assessment into everyday teaching.



FALs start with a diagnostic task, which is not graded. It gives the teacher a chance to see what the students understand (or don't). Many teachers are shocked to discover that roughly 1/3 of their students will write something like,

“Tom walked up a steep hill, got to the top and walked down for a while, until he had to walk up an even steeper hill, before he waited at the bus stop, which was flat.”

It may not be pleasant to discover this, but it's essential to know what your students are thinking!

Slide 21

Diagnostic task before the lesson

Every morning Jane walks along a straight road to a bus stop 160 meters from her home, where she catches a bus to college.
The graph shows her journey on one particular day.
Describe what may have happened.
Is the graph realistic? Why?

Time (seconds)	Distance from home (meters)
0	0
50	100
70	40
110	160

This lesson has been taught many times, and researchers have studied what students do.

The lesson plan indicates what is likely to happen when students work on the lesson. It helps prepare the teacher for things the students will say, and makes suggestions for ways to respond without simply giving the answer – using questions.

You may want to read one or two of these, but do not spend a lot of time on them.

Slide 22

Common Issue	Possible questions and prompts
Student interprets the graph as a picture E.g. as the graph goes up and down, Tom's path goes up and down.	<ul style="list-style-type: none"> If a person walked at a steady speed up and down a hill, <i>directly away from home</i>, what would the graph look like?
Student interprets graph as speed-time E.g. The student has interpreted a positive slope as speeding up and a negative slope as slowing down.	<ul style="list-style-type: none"> How can you tell if Tom is traveling away from or towards home?
Student fails to mention distance or time E.g. The student has not worked out the speed of some/all sections of the journey.	<ul style="list-style-type: none"> Can you provide more information about how far Tom has traveled during different sections of his journey?
Student fails to calculate and represent speed	<ul style="list-style-type: none"> Can you provide information about Tom's speed for all sections of his journey?
Student adds little explanation as to why the graph is or is not realistic	<ul style="list-style-type: none"> Is Tom's fastest speed realistic? Is Tom's slowest speed realistic? Why?/Why not?

Now the teacher is prepared for the lesson.

The lesson begins with a whole class discussion using this task. Which is the best story?

About 1/3 of the students typically vote for each option.

Work through option B, which represents the “taking a graph to represent a picture of the situation” mistake.

Slide 23

Lesson beginning: Which story best fits?

A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.

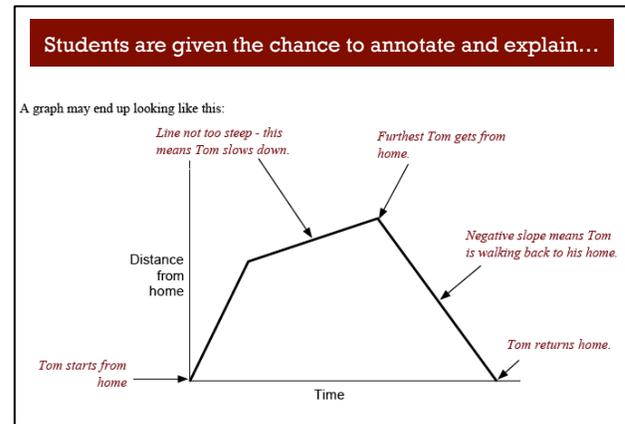
B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.

C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.

In the next segment of the lesson, teacher and class work through the graph in the previous slide, discussing what the different graph segments mean in terms of distance and time, and describing each one.

This part of the lesson serves to model the reasoning that the teacher expects of the students in the rest of the lesson.

Slide 24



Students are now given a collection of graphs and stories on cards.

They are asked to work in groups to match stories to graphs, as they prepare to make posters.

The teacher can circulate through the classroom as they do. It's easy to see where the students are making sense, where they're being challenged.

The use of shared resources like this encourages discussion among students. The teacher can listen to their reasoning, and challenge it where necessary.

Slide 25

Matching graphs and stories

		1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.	2 Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.
		3 Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.	4 Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.
		5 Tom left his home for a run, but he was unfit and gradually came to a stop!	6 Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.
		7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.	8 This graph is just plain wrong. How can Tom be in two places at once?
		9 After the party, Tom walked slowly all the way home.	10 Make up your own story!

Now another whole class discussion. The teacher asks students to think again about the situation in a different way.

To help the students make connections, the teacher shows them how they can take one of the graphs – which does not have numbers on the axes – and make it more concrete, by assigning numbers. Then they can make a table using those numbers. The table helps to make sense of the stories that might match the graph.

Slide 26

Students work on converting graphs to tables:

Whole-class discussion: Interpreting tables (15 minutes)
Bring the class together and give each student a mini-whiteboard, a pen, and an eraser. Display Slide 5 of the projector resource:

Making Up Data for a Graph

On your whiteboard, create a table that shows possible times and distances for Tom's journey.

This new representation - the table – makes things more understandable, so we now ask the students to make posters that contain triples of matching graphs, stories, and tables.

The slide has 3 of each. You might like to spend a minute matching them, talking with your neighbour.

In the lesson, there are 10 of each

Slide 27

Tables are added to the card sort...

2 Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.

1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.

6 Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120

Time	Distance
0	0
1	40
2	40
3	40
4	20
5	0

Time	Distance
0	0
1	20
2	40
3	40
4	40
5	0

And the class compares solutions together

That yields posters like this one, which they then go on to annotate, explaining WHY each triad fits together.

The students compare posters with each other, and may present them to the whole class.

Slide 28

Finally, the lesson concludes with the teacher returning the original diagnostic task and asking the students to rethink it, correcting any earlier mistakes. Students are invited to state why they have changed their minds about anything.

They are also asked to do another, similar task.

In this way, both the student and teacher can monitor what has been learned.

Slide 29

End of lesson task

Sylvia bikes along a straight road from her friend's house home, a distance of 7 miles. The graph shows her journey.

Describe what may have happened.
Include details like how fast she bikes.

So how does this lesson match up to the TRU framework?

This slide shows how it matches the Mathematics Dimension.

Its clearly important math and the lesson makes many connections.

In what follows, you want to move quickly – naming each dimension and paraphrasing what’s on the bottom of the slide. Make clear, on the run, that the FALs and TRU are completely in synch.

Slide 30

The Mathematics

How rich – connected, conceptual – is the mathematical content?

The lesson focuses on developing deep understandings of concepts like slope, and its use to describe real world phenomena; it provides opportunities to make connections across different representations (graphs, tables, stories.)

The lesson is also challenging. The whole lesson is about sense-making.

The teacher’s role is to help students interpret and translate between the different representations.

Slide 31

Cognitive Demand

To what extent are students supported in grappling with and making sense of mathematical concepts?

The card sort and poster activities provide plenty of room for sense making – *if* the students are gently supported when they need it. (Remember the list of support questions)

No curricular materials can do the complete job with regard to access and agency/identity – this takes hard, sensitive work on the part of the teacher.

However, the shared resources and the lesson structure provides opportunities for all students to collaborate and reason together.

Slide 32

Access to Mathematical Content

To what extent does the teacher support access to the content of the lesson for all students?

The classroom structures – which include whole group conversations, small group work, and student poster presentations – provide *opportunities* for teachers to support every student in engaging meaningfully with the mathematics. But . . . this takes hard work, even with the opportunities.

Again, this isn't guaranteed to happen. But if the teacher uses these materials in the ways promoted in the teacher guide, then students will be involved in explaining to one another and engaging in each other's reasoning.

Slide 33

Agency, Ownership, and Identity

To what extent are students the source of ideas and discussion of them? How are student contributions framed?

The classroom structures – which include whole group conversations, small group work, and student poster presentations – provide *opportunities* for teachers to support every student in building powerful mathematical identities. But . . . this takes hard work, even with the opportunities.

These are known as formative assessment lessons for a reason!

- The initial task exposes current thinking.
- The tasks are designed to promote discussion about common misconceptions.
- Students are invited to assess the reasoning of other students.
- This process moves students' reasoning forward – the lesson provides diagnosis leading into treatment.

So you see, in all five dimensions, these lessons and TRU are in perfect harmony!

Slide 34

Formative Assessment

To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?

They're known as
Formative Assessment Lessons
for a reason...



Now the question: so, everything fits together – but, does it work?

The Gates Foundation supported professional development in various places, including Kentucky.

Professional development was provided by the Mathematics Design Collaborative, which helped teachers implement these lessons in the ways the designers intended.

The Foundation hired independent evaluators to examine the impact of them.

Source:

https://www.cse.ucla.edu/products/policy/PB_13.pdf

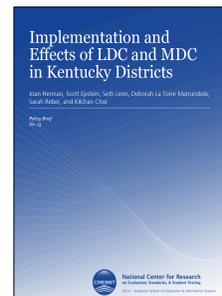
Slide 35

Is this type of lesson effective?

The National Center for Research on Evaluation, Standards and Student Testing (CRESST)

assessed the effectiveness of the Math Design Collaborative's (MDC) use of Classroom Challenges in 9th Grade Kentucky classrooms.

What did they find?



Here is what they found.

That's almost unbelievable. How could it be?

What makes the lessons effective is that the teacher shifts to a much more student-centered pedagogy – “meeting students where they are” and helping them build up their understandings. This is a skill that transfers to regular instruction – and when it does, students learn more.

So, the impact of the lessons goes beyond them – teachers are moving to a "classroom culture" more in line with TRU.

Slide 36

Is this type of lesson effective?

- “For MDC, participating teachers were expected to implement between four and six Challenges, meaning that **students were engaged only 8-12 days of the school year**
- Nonetheless, the studies found **statistically significant learning effects... the approximate equivalent of 4.6 months for MDC**. Given their contexts of early implementation and limited dosage, these small effects are noteworthy.”

The next tool is designed to help teachers refine their teaching, more in line with TRU.

This tool is reproduced in full in **Handout 4**.

Slide 37

Tool (b) Planning and Reflection

- The TRU Math Conversation Guide.
- The TRU dimensions become arenas for teachers to reflect on their own teaching:
 - in planning,
 - in reflecting on how things have gone
 - in thinking about next steps.

TRU Math Conversation Guide:
A Tool for Teacher Learning and Growth¹

This TRU Math Conversation Guide is a product of The Algebra Teaching Study (NSF Grant DRL-0908153 to Ph. Alan Schoenfeld, U.C. Berkeley, and NSF Grant DRL-0908152 to Ph. Robert Fyfe, Michigan State University), and of The Mathematics Assessment Project (IIT and Michigan State Foundation Grant QP13342 to Ph. Alan Schoenfeld, U.C. Berkeley, and Hugh Burkhardt and Malcolm Swan, The University of Nottingham).

A companion document, the TRU Math Conversation Guide: Module A: Contextual Algebraic Tasks, supports the depth requirements of algebraic thinking with 10 focus on complex modeling and applications problems. Module A: Contextual Algebraic Tasks is the first of a series of context-specific conversation guides aimed at supporting classroom engagement with complex, rigorous mathematical ideas. The TRU Math Conversation Guide Modules will all be available at <http://dx.doi.org/10.21203/rs.1.rs1000000/v1>.

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²For an online list of all guides, visit the conversation guide list page that will link to the next page for each guide. We also welcome comments and suggestions for improvements. Please contact cbaldinger@berkeley.edu and alouie@msu.edu with any feedback.

The idea is to take core questions related to the five dimensions...

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Start with the core questions:

- The Mathematics**
How do mathematical ideas from this unit/course develop in this lesson/lesson sequence?
- Cognitive Demand**
What opportunities do students have to make their own sense of mathematical ideas?
- Access to Mathematical Content**
Who does and does not participate in the mathematical work of the class, and how?
- Agency, Ownership, and Identity**
What opportunities do students have to explain their own and respond to each other's mathematical ideas?
- Formative Assessment**
What do we know about each student's current mathematical thinking, and how can we build on it?

And turn them into sets of questions that can be used for planning and reflection – perhaps by oneself, but even better with colleagues.

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... and expand them.

- **Before a lesson, you can ask:**
 - How can I use the five dimensions to enhance my lesson planning?
- **After a lesson, you can ask:**
 - How well did things go? What can I do better next time?
- **Planning next Steps, you can ask:**
 - How can I build on what I've learned?

Now refer participants to **Handout 4**

Looking at the TRU conversation guide, I'm going to flip through the guide to show you what it looks like, and make a quick stop at "access" to illustrate the kind of conversations its designed to support.

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Looking at the conversation guide

The image shows a stack of pages from the TRU Math Conversation Guide. The top page is titled "Access to Mathematical Content" and includes a "Formative Assessment" table with questions like "What opportunities exist for each student to participate in the mathematical work of the class?" and "How can we create opportunities for each student to participate in the mathematical work of the class?". Below the table is a "Planning Next Steps" section. Other pages visible in the stack include "The Mathematics", "Cognitive Demand", "Agency, Authority, and Identity", and another "Formative Assessment" page.

This is what the expansion for access looks like. Let's take a closer look at this one. I'm going to read some of the "think abouts" down at the bottom.

"Think about:

- The range of ways students can and do participate in the mathematical work of the class.
- Which students participate in which ways.
- Which students are most active when, and how we can create opportunities for more students to participate more actively.
- What opportunities various students have to make meaningful mathematical contributions."

Now imagine teachers and coaches planning together, watching each other teach and debriefing using these ideas.

What's critically important is to make thinking like this a habit, so you think about these issues all the time – in planning, in teaching, in reflecting.

We've built distilled versions that are useful in watching videos in PD, or for keeping "at the top of your head" for reflection.

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Access to Mathematical Content

Core Question: Who does and does not participate in the mathematical work of the class, and how

All students should have access to opportunities to develop their own understandings of rich mathematics, and to build productive mathematical identities. For any number of reasons, it can be extremely difficult to provide this access to everyone, but that doesn't make it any less important! We want to challenge ourselves to recognize who has access and when. There may be mathematically rich discussions or other mathematically productive activities in the classroom—but who gets to participate in them? Who might benefit from different ways of organizing classroom activity?

Access to Mathematical Content		
Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities exist for each student to participate in the mathematical work of the class?	Who did and didn't participate in the mathematical work of the class, and how?	How can we create opportunities for each student to participate in the mathematical work of the class?
<p><i>Think about:</i></p> <ul style="list-style-type: none"> ○ The range of ways students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard, manipulating symbols, making diagrams, interpreting graphs, using manipulatives, connecting different strategies, etc.). ○ Which students participate in which ways. ○ Which students are most active when, and how we can create opportunities for more students to participate more actively. ○ What opportunities various students have to make meaningful mathematical contributions. ○ Language demands and the development of students' academic language. ○ How norms (or interactions, or lesson structures, or task structure, or particular representations, etc.) facilitate or inhibit participation for particular students. ○ What teacher moves might expand students' access to meaningful participation (such as modeling ways to participate, providing opportunities for practice, holding students accountable, pointing out students' successful participation). ○ How to support particular students we are concerned about (in relation to learning, issues of safety, participation, etc.). 		

The third and final tool we will consider is the observation of classrooms: **Handout 5**

This can be done in many ways, depending on the purpose. However, TRU has been modified to suit these different purposes.

Lets look quickly at three.

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Tool (c) Ways to Observe Classrooms

Here are three ways of observing:

- Observe as a teacher.
- Observe, as a student.
- Observe as a researcher.

One way to make observations is to start with an open mind and brief sets of questions, like those in this slide. This is a distilled version of TRU, suitable for everyday use by a teacher.

In fact, you can have an observation sheet with five columns, with the questions for each dimension at the head of each column.

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Observe as a teacher	
The Mathematics	<ul style="list-style-type: none"> Are students learning important mathematics? Are opportunities made for meaningful connections?
Cognitive Demand	<ul style="list-style-type: none"> How long do students spend on each prompt? Do they engage in productive struggle? Do teacher questions invite explanations or answers?
Access to Mathematical Content	<ul style="list-style-type: none"> Are there multiple ways to get involved productively? Does the teacher ask a range of students to respond?
Agency, Authority, and Identity	<ul style="list-style-type: none"> Who explains most: the teacher or the students? Do the students give extended explanations?
Formative Assessment	<ul style="list-style-type: none"> Does the teacher follow up on student responses? Does the teacher vary the lesson in the light of student responses?

Even better, you can frame the observations from the point of view of the student.

Imagine their experiences, with summaries organized in those five columns.

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Observe as if you were a student	
The Mathematics	<ul style="list-style-type: none"> What's the big mathematical idea in this lesson? How does it connect to what I already know?
Cognitive Demand	<ul style="list-style-type: none"> How long am I given to think, and to make sense of things? What happens when I get stuck? Am I invited to explain things, or just give answers?
Access to Mathematical Content	<ul style="list-style-type: none"> Do I get to participate in meaningful math learning? Can I hide or be ignored?
Agency, Authority, and Identity	<ul style="list-style-type: none"> Do I get to explain, to present my ideas? Are they built on? Am I recognized as being capable and able to contribute in meaningful ways?
Formative Assessment	<ul style="list-style-type: none"> Do classroom discussions include my thinking? Does instruction respond to my thinking and help me think more deeply?

Of course, TRU also has a research tool that lists classroom activities in order of increasing richness along each dimension. That's in this slide.

What we care about is the developmental trajectory along each dimension – if your classroom looks like it's in the middle along some dimension, what activities might you consider to do better?

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Observe as if you were a researcher				
The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Formative Assessment
How rich – conceptual, connected – is the mathematical content?	To what extent are students supported in grappling with and making sense of mathematical concepts?	To what extent does the teacher support access to the content of the lesson for all students?	To what extent are students the source of ideas and discussion of them? How are student contributions framed?	To what extent to students' mathematical thinking surfaced? To what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?
1 Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement in key practices such as reasoning and problem solving.	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue.	The teacher initiates conversations; students' speech turns are short (one sentence or less), and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2 Activities are primarily skills-oriented, with cursory connections between procedures, concepts and content (where appropriate) and minimal attention to key practices.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but the student proposes, the teacher disposes; in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3 Classroom activities support meaningful connections between procedures, concepts and content (where appropriate) and provide opportunities for engagement in key practices.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation. OR what appear to be established participation structures result in such engagement.	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, AND/OR students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

BUT, we recognize that any scoring framework, even if developed for research, can be used to score teachers.

We don't like that – nor is it reasonable.

To be valid and fair, any evaluation needs many dimensions and many samples.

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Misusing TRU

- You can use also use TRU to grade teachers ... we can't stop you.
- But he most important use of a yardstick is to measure growth.



This is where you can find many of our resources.

The big idea is to build professional learning communities that work toward realizing the goals in TRU.

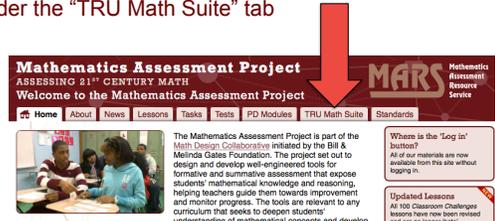
It's worth looking at the MathNIC website for information about other tools that complement this.

Over the next year we hope to develop further tools that can help.

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Resources

The TRU Math Suite and supporting documents are available on
The Mathematics Assessment Project web site:
<http://map.mathshell.org/>
Under the "TRU Math Suite" tab



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Q&A

Questions?

Comments?

Suggestions?

Now's the time for Q&A.

We have time for a brief discussion

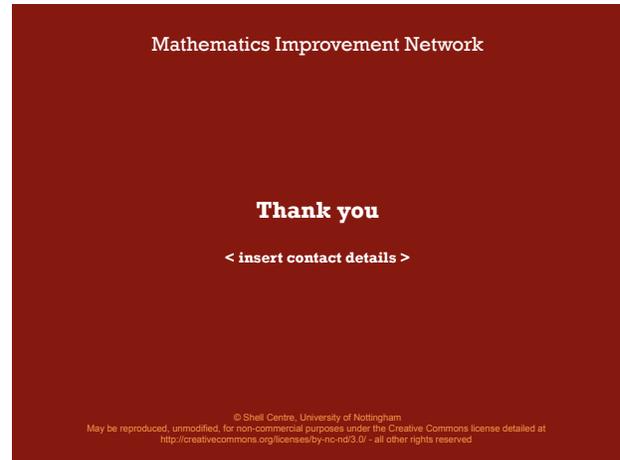
At the end, if you are willing

Feel free to contact me later.

Or, perhaps, name another person

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Customize this slide with your contact details



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