Teaching Mathematics for Robust Understanding
What makes a mathematically powerful classroom?

Handouts

Contents

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Handout 1: Establishing a focus

If you had 5 things to focus on in order to improve mathematics teaching, what would they be?

1.

2.

3.

4.

5.

Why do you think these are the right things?

Handout 2: Observations from the videos

Make notes on your observations below.
Try to imagine the lesson from the point of view of the students.

Video 1: A lesson on finding angles

Video 2: The Border Tiles lesson

Video 3: Fractions, decimals, percents

Handout 3: TRU Math Framework

The Five Dimensions of Powerful Mathematics Classrooms

The Mathematics	Cognitive Demand	Equitable Access to Content	Agency, Ownership, and Identity	Formative Assessment
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' disciplinary development. Students should be able to engage in sense making and "productive struggle."*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core content being addressed by the class. No matter how rich the content being discussed, for example, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to "walk the walk and talk the talk," building on each other's ideas, in ways that contribute to their development of agency (the willingness to engage) and ownership over the content, resulting in positive identities as thinkers and learners.*	*The extent to which classroom activities elicit student thinking and subsequent instruction responds to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.*

Handout 4: TRU Math Conversation Guide

The Mathematics

Core Questions: How do mathematical ideas from this unit/course develop in this lesson/lesson sequence? How can we create more meaningful connections?

Students often experience mathematics as a set of isolated facts, procedures and concepts, to be rehearsed, memorized, and applied. Our goal is to instead give students opportunities to experience mathematics as a coherent and meaningful discipline. This means identifying the important mathematical ideas behind facts and procedures, highlighting connections between skills and concepts, and relating concepts to each other—not just in a single lesson, but also across lessons and units. It also means engaging students with centrally important mathematics in an active way, so that they can make sense of concepts and ideas for themselves and develop robust networks of understanding.

The Mathematics

Pre-observation	Reflecting After a Lesson	Planning Next Steps
How will important mathematical ideas develop in this lesson and unit?	How did students actually engage with important mathematical ideas in this lesson?	How can we connect the mathematical ideas that surfaced in this lesson to future lessons?

Think about:

- The mathematical goals for the lesson.
- What connections exist among important ideas in this lesson and important ideas in past and future lessons.
- How math procedures in the lesson are justified and connected with important ideas.
- How we see/hear students engage with mathematical ideas during class.
- Which students get to engage deeply with important mathematical ideas.
- How we could create opportunities for more students to engage more deeply with mathematical ideas.

Cognitive Demand

Core Questions: What opportunities do students have to make their own sense of mathematical ideas? How can we create more opportunities?

We want students to engage authentically with important mathematical ideas, not simply receive knowledge. This requires students to engage in productive struggle. They need to be supported in these struggles so that they aren't lost, but at the same time, support should maintain students' opportunities to grapple with important ideas and difficult problems. Finding a balance is difficult, but our goal is to help students understand the challenges they confront, while leaving them room to make their own sense of those challenges.

Cognitive Demand

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities will students have to make their own sense of important mathematical ideas?	What opportunities did students have to make their own sense of important mathematical ideas?	How can we create more opportunities for students to make their own sense of important mathematical ideas?

Think about:

- What opportunities exist for students to struggle with mathematical ideas.
- How students' struggles may support their engagement with mathematical ideas.
- How the teacher responds to students' struggles and how these responses support students to engage without removing struggles.
- What resources (other students, the teacher, notes, texts, technology, manipulatives, various representations, etc.) are available for students to use when they encounter struggles.
- What resources students actually use and how they might be supported to make better use of resources.
- Which students get to engage deeply with important mathematical ideas.
- How we could create opportunities for more students to engage more deeply with mathematical ideas.
- What community norms seem to be evolving around the value of struggle and mistakes.

Access to Mathematical Content

Core Questions: Who does and does not participate in the mathematical work of the class, and how? How can we create more opportunities for each student to participate meaningfully?

All students should have access to opportunities to develop their own understandings of rich mathematics, and to build productive mathematical identities. For any number of reasons, it can be extremely difficult to provide this access to everyone, but that doesn't make it any less important! We want to challenge ourselves to recognize who has access and when. There may be mathematically rich discussions or other mathematically productive activities in the classroom—but who gets to participate in them? Who might benefit from different ways of organizing classroom activity?

Access to Mathematical Content

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities exist for each student to participate in the mathematical work of the class?	Who did and didn't participate in the mathematical work of the class, and how?	How can we create opportunities for each student to participate in the mathematical work of the class?

Think about:

- What range of ways students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard; manipulating symbols, making diagrams, interpreting graphs, using manipulatives, connecting different strategies, etc.).
- Which students participate in which ways.
- Which students are most active when, and how we can create opportunities for more students to participate more actively.
- What opportunities various students have to make meaningful mathematical contributions.
- Language demands and the development of students' academic language.
- How norms (or interactions, or lesson structures, or task structure, or particular representations, etc.) facilitate or inhibit participation for particular students.
- What teacher moves might expand students' access to meaningful participation (such as modeling ways to participate, providing opportunities for practice, holding students accountable, pointing out students' successful participation).
- How to support particular students we are concerned about (in relation to learning, issues of safety, participation, etc.).

Agency, Authority, and Identity

Core Questions: What opportunities do students have to see themselves and each other as powerful doers of mathematics? How can we create more of these opportunities?

Many students have negative beliefs about themselves and mathematics, for example, that they are "bad at math," or that math is just a bunch of facts and formulas that they're supposed to memorize. Our goal is to support all students—especially those who have not been successful with mathematics in the past—to develop a sense of mathematical agency and authority. We want students to come to see themselves as mathematically capable and competent—not by giving them easy successes, but by engaging them as sense-makers, problem solvers, and creators of mathematical ideas.

Agency, Authority, and Identity

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What opportunities exist in the lesson for students to explain their own and respond to each other's mathematical ideas?	What opportunities did students have to explain their own and respond to each other's mathematical ideas?	What opportunities can we create in future lessons for more students to explain their own and respond to each other's mathematical ideas?

Think about:

- Who generates the mathematical ideas that get discussed.
- Who evaluates and/or responds to others' ideas.
- How deeply students get to explain their ideas.
- How the teacher responds to student ideas (evaluating, questioning, probing, soliciting responses from other students, etc.).
- How norms around students' and teachers' roles in generating mathematical ideas are developing.
- How norms around what counts as mathematics (justifying, experimenting, practicing, etc.) are developing.
- Which students get to explain their own and respond to others' ideas in a meaningful way.
- Which students seem to see themselves as powerful mathematical thinkers right now.
- How we might create opportunities for more students to see themselves and each other as powerful mathematical thinkers.

Uses of Assessment

Core Questions: What do we know about each student's current mathematical thinking? How can we build on it?

We want instruction to be responsive to students' actual thinking, not just our hopes or assumptions about what they do and don't understand. It isn't always easy to know what students are thinking, much less to use this information to shape classroom activities—but we can craft tasks and ask purposeful questions that give us insights into the strategies students are using, the depth of their conceptual understanding, and so on. Our goal is to then use those insights to guide our instruction, not just to fix mistakes but to integrate students' understandings, partial though they may be, and build on them.

Uses of Assessment

Pre-observation	Reflecting After a Lesson	Planning Next Steps
What do we know about each student's current mathematical thinking, and how does this lesson build on it?	What did we learn in this lesson about each student's mathematical thinking? How was this thinking built on?	Based on what we learned about each student's mathematical thinking, how can we (1) learn more about it and (2) build on it?

Think about:

- What opportunities exist for students to develop their own strategies and approaches.
- What opportunities exist for students to share their mathematical ideas and reasoning, and to connect their ideas to others'.
- What different ways students get to share their mathematical ideas and reasoning (writing on paper, speaking, writing on the board, creating diagrams, demonstrating with manipulatives, etc.).
- Who students get to share their ideas with (e.g., a partner, the whole class, the teacher).
- How students are likely to make sense of the mathematics in the lesson and what responses might build on that thinking.
- What things we can try (e.g., tasks, lesson structures, questioning prompts such as those in FALs) to surface student thinking, especially the thinking of students whose mathematical ideas we don't know much about yet.
- What we know and don't know about how each student is making sense of the mathematics we are focusing on.
- What opportunities exist to build on students' mathematical thinking, and how teachers and/or other students take up these opportunities.

Handout 5: Ways to observe classrooms

Observe as a teacher

The Mathematics	• Are students learning important mathematics? • Are opportunities made for meaningful connections?
Cognitive Demand	• How long do students spend on each prompt? • Do they engage in productive struggle? • Do teacher questions invite explanations or answers?
Access to Mathematical Content	• Are there multiple ways to get involved productively? • Does the teacher ask a range of students to respond?
Agency, Authority, and Identity	• Who explains most: the teacher or the students? • Do the students give extended explanations?
Formative Assessment	• Does the teacher follow up on student responses? • Does the teacher vary the lesson in the light of student responses?

Observe as if you were a student

The Mathematics	• What's the big mathematical idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	• How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Access to Mathematical Content	• Do I get to participate in meaningful math learning? • Can I hide or be ignored?
Agency, Authority, and Identity	• Do I get to explain, to present my ideas? Are they built on? • Am I recognized as being capable and able to contribute in meaningful ways?
Formative Assessment	• Do classroom discussions include my thinking? • Does instruction respond to my thinking and help me think more deeply?

Observe as if you were a researcher

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Formative Assessment
	How rich – conceptual, connected – is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to the content of the lesson for all students?*	*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*
1	Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement in key practices such as reasoning and problem solving.	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue.	The teacher initiates conversations. Students' speech turns are short (one sentence or less), and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	Activities are primarily skills-oriented, with cursory connections between procedures, concepts and contexts (where appropriate) and minimal attention to key practices.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for engagement in key practices.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; **OR** what appear to be established participation structures result in such engagement.	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.